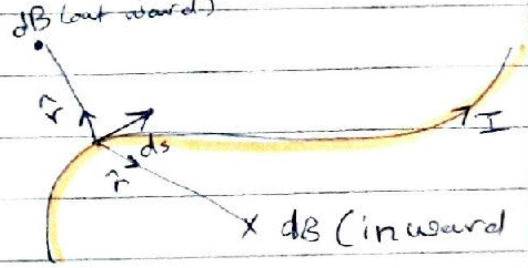
29 - Magnetic fields Due to Current.

- Current I produces Magnetic field ($\vec{B}$) By using Biot Savart law and/or Amperes law.

⇒ Biot - Savart law:-

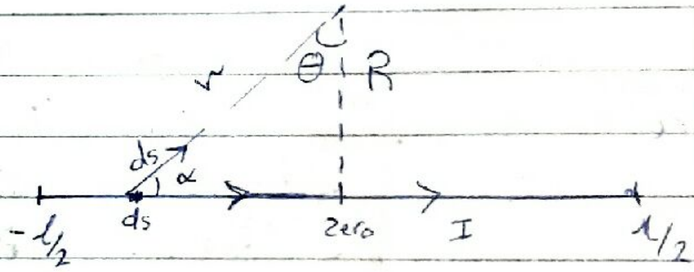
- $dB = \frac{\mu_0}{4\pi} i \frac{d\vec{s} \times \hat{r}}{r^2}$; $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Tesla} \cdot m}{A}$ (permeability constant of air).

$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2}$, $d\vec{B} \perp \vec{r}$, $d\vec{B} \perp d\vec{s}$



1 $\vec{B}$ due to I in astraight wire:

wire
→ length = l
→ Current = I
→ Find B adistance (R) above the mid Point ?!



$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2}$$

$$B = \int_{-l/2}^{l/2} \frac{\mu_0\, i \sin\theta\, ds}{4\pi r^2} = \frac{\mu_0 i}{4\pi} \int_{-l/2}^{l/2} \frac{\cos\theta\, ds}{r^2}$$

⇒ $x = R\tan\theta$
$ds = dx$
$dx = R\sec^2\theta\, d\theta$

⇒ $\cos\theta = \frac{R}{r}$

$r = \frac{R}{\cos\theta} = R\sec\theta$

$$B = \frac{\mu_0 i}{4\pi} \int_{-\theta_0}^{\theta_0} \frac{\cos\theta(R\sec^2\theta\, d\theta)}{R^2 \sec^2\theta} = \frac{\mu_0 i}{4\pi} \int_{-\theta_0}^{\theta_0} \cos\theta\, d\theta = \frac{\mu_0\, i \sin\theta_0}{2\pi R}$$

• find B due to I in infinit wire ?!

as $l \longrightarrow \infty$, $\theta_0 \longrightarrow 90^\circ$

$$B = \frac{\mu_0 i}{2\pi R}$$

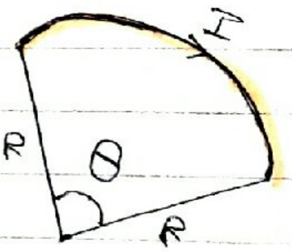
• Find B due to I in asemi-infinit wire ?

$$B = \frac{\mu_0 i}{4\pi R}\left(\sin\overset{zero}{\theta_0} - \sin\overset{90}{\theta_0}\right) \Longrightarrow B = \frac{\mu_0 i}{4\pi R}$$

$\boxed{2}$ Magnetic field at the Center of circular Arc of current

circular Arc
- current = I
- radius = R
- Angle At the center = θ
- length = $R\theta$ = $R\theta$ rad
- find B at the center ?

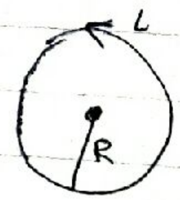
$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\,\sin 90}{R^2}$$



$$B = \frac{\mu_0 i}{4\pi R^2}\int ds = \frac{\mu_0 i (R\theta)}{4\pi R^2}$$

$$B = \frac{\mu_0 i \theta}{4\pi R}$$

* For complete Circle , $\theta = 2\pi$

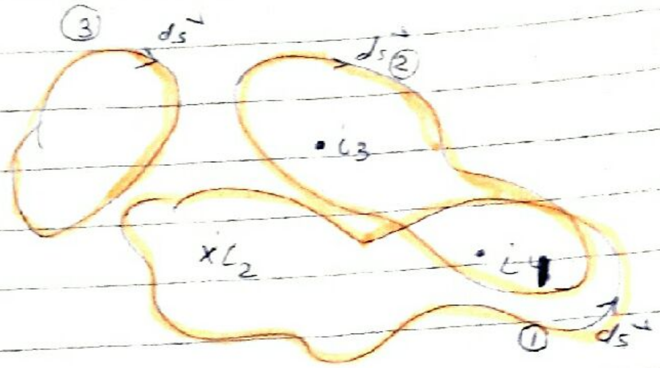


$$B = \frac{\mu_0 i}{2R}$$

* For N circle $\Longrightarrow$ $B = \frac{\mu_0 N i}{2R}$

⇒ Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, i_{enc}$$



$$\oint_1 \vec{B} \cdot d\vec{s} = \mu_0 (i_1 - i_2)$$

$$\oint_2 \vec{B} \cdot d\vec{s} = \mu_0 (i_1 + i_3)$$

$$\oint_3 \vec{B} \cdot d\vec{s} = \text{zero}$$

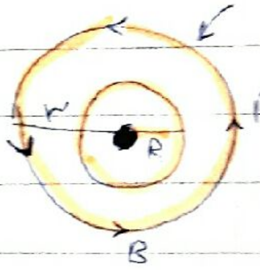
• التيارات الموجودة خارج المسار المغلق تؤثر بمجال على المسار ولكن تكاملها بالنسبة للمساحة يساوي صفراً.

• Amperes law is useful to calculat $\vec{B}$ due to I in asystem Having High symmetry.

1 $\vec{B}$ due to I in along Straight wire:

a very long straight wire →
- radius = R
- current = I
- current density = $\frac{I}{\pi R^2}$

Amperian loop



(a) find B at $r \geq R$?

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, i_{enc}$$

$$B \oint \cos 0 \, ds = \mu_0 I$$

$$B \, 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \, , \; r \geq R \implies B_{surface} = \frac{\mu_0 I}{2\pi R}$$

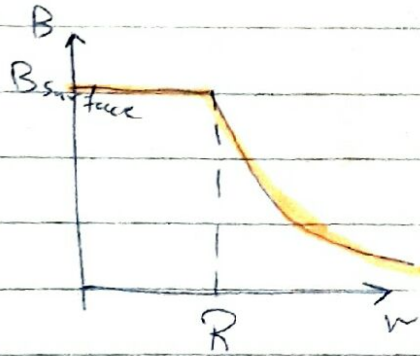
(b) find $\vec{B}$ at $r \le R$?

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I_{enc}$$

$$B\, 2\pi r = \mu_0 (J \times \pi r^2)$$

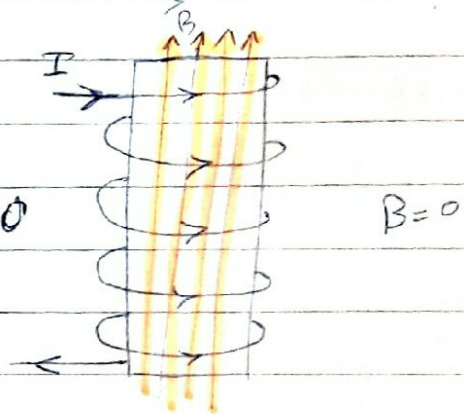
$$B\, 2\pi r = \mu_0 \left(\frac{I}{\pi R^2} \cdot \pi r^2 \right)$$

$$B = \frac{\mu_0 I r}{2\pi R^2} \quad , \quad r \le R$$



2 Magnetic field due to I in a solenoid?

ideal solenoid
- length (l)
- radius (R)
- $l \gg R$
- nember of terms = N
- Current = I



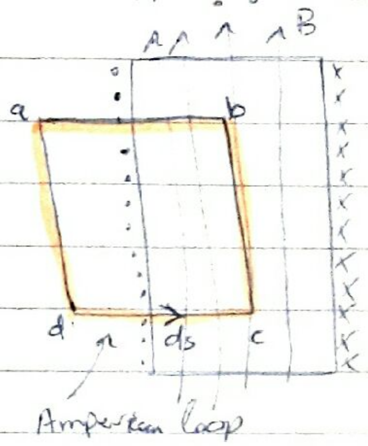
• $\vec{B}$ inside the solenoid is uniform, find it?!

$$\int_a^d \vec{B} \cdot \vec{ds} + \int_d^c \vec{B} \cdot \vec{ds} + \int_c^d \vec{B} \cdot \vec{ds} + \int_b^a \vec{B} \cdot \vec{ds} = \mu_0 I_{enc}$$

$$0 + 0 + B\, l_{cb} + 0 = \mu_0 I_{enc}$$

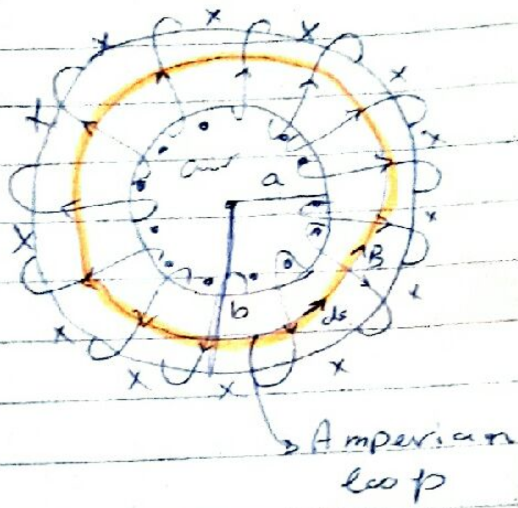
$$B\, l_{cb} = \mu_0 N I \Rightarrow \mu_0 \frac{N}{l_{cb}} I = B$$

$$\Rightarrow B = \mu_0 n I$$



3 B of a Toroid :-

Toroid
- inner radius = a
- outer radius = b
- number of turns = N
- Current = I

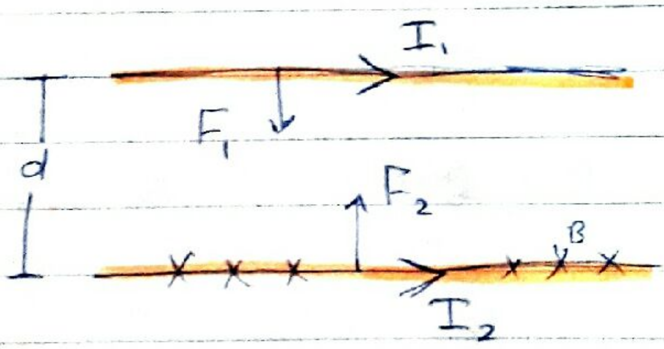


$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$B \, 2\pi r = \mu_0 N I$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}} \quad , \quad b \geq r \geq a$$

⟹ Magnetic force between 2 parallel Currents :



- $B_1 = \dfrac{\mu_0 I_1}{2\pi d}$
- $F_2 = I_2 \vec{l}_2 \times \vec{B}_1 = I_2 \, l_2 \left(\dfrac{\mu_0 I_1 N}{2\pi d}\right)$

$$F_2 = \frac{I_2 I_1 \, l_2 \, \mu_0 N}{2\pi d} \quad (N)$$

$$\frac{F_2}{l_2} = \frac{I_2 I_1 \, \mu_0 N}{2\pi d} \quad (N/m)$$